Accurate microwave permittivity data of some reference liquids

The basic data given here were presented at the 6th ISEMA Conference in Weimar, May–June 2005. New measurements using complete multi-step numerical modelling of a dual resonant cavity at about 920 MHz and 2230 MHz were presented. Water at + 20°C is used as reference for the calculations. Due to the high resolution and considerations of various error sources, the resulting accuracy becomes high and allows the construction of improved empirical formulæ for the Debye relaxation behaviour.

The Debye relaxation equation

This is $\varepsilon = \varepsilon_\infty + \dfrac{\varepsilon_s - \varepsilon_\infty}{1 + j \cdot \left(\dfrac{f}{f_D}\right)}$ (1)

where

f is the frequency;
ε is the complex (relative) permittivity $\varepsilon' - j\varepsilon''$;
ε_s is the static permittivity ($f \to 0$);
ε_∞ is the (fictitious) permittivity for $f \to \infty$; and
f_D is the Debye relaxation frequency

A distribution parameter α may be introduced, but there is a consensus that this is not applicable to water.

With known values of ε_s, ε_∞ and f_D one may calculate the complex ε for any frequency f, using equation (1), on a calculator handling complex numbers.

Basic assumptions

The measurement method relies on using a given set of ε_s, ε_∞ and f_D of water at +20 °C, to obtain the sample tube diameter and certain cavity conductivity variation corrections. Based on a detailed literature survey, the following basic data was used:

$$\varepsilon_s = (80,30 \pm 0,10) \quad \varepsilon_\infty = (5,0\pm0,2) \quad \alpha = 0$$
$$f_D = (16,8 \pm 0,6)\ \text{GHz} \qquad t = 20,00\ °C \qquad (2)$$

The error limits given above represent the true absolute data with a very high certainty. The nominal values are used in the following.

The accurate measurements were made in two frequency bands: about 920 and 2230 MHz. The accuracy is therefore *best from very low frequencies up to about 3 GHz.*

The temperature range was in most cases from about −5 °C to +40 °C, with a *temperature inaccuracy of less than ±0,2K.*

Error limits are given for each liquid.

Pyridine – C_5H_5N – Debye relaxation data

Manufacturer: Merck; purity >99,5 %, with <0,2 % α-picoline and <0,1 % water.

$\varepsilon_s(t) = 1/(0,068078+t \cdot 0,0003277)$ *(t in °C)*   (3a)

$f_D(t) = -179,297+199,70 \cdot \exp(t/461,00)$ *(t in °C)* (3b)

$\varepsilon_\infty = 2,30$ *(no temperature dependence)*   (3c)

The absolute error in ε_s between +10 and +30 °C is < ±0,15 units, and < ±0,3 GHz in f_D. The resulting errors in ε' in the frequency range 0 to 3 GHz is the same as in ε_s. For ε'', the error may be up to 2 % .

As an example of calculated data at +20 °C, ε = 13,32–j0,923 at and 2450 MHz, and 13,39–j0,347 at 915 MHz.

Dimethylformamide (DMF) – $HCON(CH_3)_2$ – Debye relaxation data

Manufacturer: Merck; purity >99,9 %, with <0,02 % water.

$\varepsilon_s(t) = 14,754+27,246 \cdot \exp(-t/133,62)$ *(t in °C)* (4a)

$f_D(t) = 1,303+9,747 \cdot \exp(t/56,012)$ *(t in °C)*   (4b)

$\varepsilon_\infty = 4,00$ *(no temperature dependence)*   (4c)

The absolute error in ε_s and ε' between +10 and +30 °C is < ±0,3 units, in the frequency range 0 to 3 GHz. For ε'', the error may be up to 4 % .

As an example of calculated data at +20 °C, ε = 37,35–j5,36 at and 2450 MHz, and 38,09–j2,05 at 915 MHz.

Mixture of 54,0 % methanol + 46 % water by weight – Debye relaxation data

MeOH manufacturer: Merck; purity >99,8 %, with <0,05 % water and <0,1 % ethanol.

$\varepsilon_s(t) = 65,56-2,656 \cdot \sqrt{t}$ *(t in °C)*   (5a)

$f_D(t) = \exp(1,0055+0,02656 \cdot t)$ *(t in °C)*   (5b)

$\varepsilon_\infty = 5,00$ *(no temperature dependence)*   (5c)

The absolute error in ε_s and ε' between +10 and +30 °C is < ±0,4 units, in the frequency range 0 to 3 GHz. For ε'', the error may be up to 5 % .

As an example of calculated data at +20 °C, ε = 43,10–j20,08 at and 2450 MHz, and 51,87–j9,22 at 915 MHz.